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1 Disclaimer

This note is being posted at the request of the collaboration, although it is not complete and a full set of cross checks have not been performed. This version of the note is only intended to inform other members of the collaboration of a potential effect which is under investigation.

2 Introduction

The MICE experiment seeks to measure the emittance of a beam before and after cooling. One of the important issues for this experiment is the effect of backgrounds on the emittance measurement. In this context, it is necessary to define a figure of merit for the quality of the measurement. This figure of merit can be used to study the degradation in performance as a function of background intensity and also to compare different tracking systems under a variety of conditions.

A possible figure of merit is the emittance resolution, defined in terms of the measured and reconstructed emittances for a large number of runs. An important associated quantity is the bias in the measurement, which is the mean of the difference between true and reconstructed emittance.

A definition for emittance must be chosen; in this case the RMS emittance was chosen, as defined in Equation 3. Some properties of this emittance have been studied, and a few curious results obtained.

For these studies, a group of particles were generated which filled the acceptance of a solenoidal channel 30 cm in diameter. The particles were uniformly distributed in position across the entrance of the channel and the momentum direction for each was chosen randomly with uniform probabilty

r	radius of helix
x_0	x of the axis of helix
y_0	y of the axis of helix
ϕ_0	$\phi(z=0)$
α	$\Delta\phi/\Delta z$

Table 1: Defintion of Track Parameters

on the forward pointing unit hemisphere in momentum space. Several different energy distributions were used, and will be described as they become relevant.

This note deals with a particular property of the emittance under these circumstances, specifically how the emittance changes along the channel. This is studied by taking the true track parameters, as defined in Equations 1 and 2, and propogating the particles to different positions in z. It is emphasized that this effect has nothing to do with measurement errors. It is a property only of the RMS emittance in a perfect solenoid.

3 Definitions

Because the beam is propogating in a perfect solenoid, the logical choice for track parameters are the parameters of a helix. The parameters are defined in Table 1 and the equations of a track are given in 1 and 2. The RMS emittance is defined in Equation 3.

$$x = r\sin(\alpha z + \phi_0) + x_0 \tag{1}$$
$$y = r\cos(\alpha z + \phi_0) + y_0 \tag{2}$$

$$\epsilon^2 = \left| \begin{pmatrix} <x^2> - <x>^2 & <xx'> - <x><x'> & <xy> - <x><y> & <x><y'> - <x><y'> \\ <xx'> - <x><x'> & <(x')^2> - <x'>^2 & <x'y> - <x'><y> & <x'><y'> - <x'><y'> \\ <xy> - <x><y> & <x'y> - <x'><y> & <y^2> - <y>^2 & <yy'> - <y><y'> \\ <xy'> - <x><y'> & <x'y'> - <x'><y'> & <yy'> - <y><y'> & <(y')^2> - <y'>^2 \end{pmatrix} \right| \tag{3}$$

4 Monte Carlo Results

The Monte Carlo results shown here were generated using GEANT. The results do not rely on GEANT, however. The true position and momentum of beam particles were taken from GEANT at a plane perpindicular to the z axis, and all results are found by propagating these particles to a different z position using Equations 1 and 2. The field in the simulation was an ideal solenoidal field of 5 Tesla parallel to the z axis. The event selection criteria listed in Tablewere applied when Figures 1 and 2 were created.

Figure 1 shows the RMS emittance as a function of z for a mono energetic beam with momentum xxx, for several sample sizes. The samples are inclusive; each sample is a subset of the larger ones.

An interesting feature is the period of the oscillation. It is aproximately zzzz for all sample sizes. This seems to correspond to the Larmor frequency of the muons in this field, which is...

Figure 2 shows the emittance distribution for a beam with a flat momentum distribution from xxx to yyy. In this case we see that the different Larmor frequencies beat against each other, yielding a complicated curve. The possibility that the event selection causes the effect has been investigated. Figure 3 shows the z distribution of emittance for the same sample as Figure 2, but with no event selection criteria applied. The effect is still clearly present.

5 Analytical Results

In order to better understand the effect, the analytical calculation of the emittance as defined in Equation 3 using the track equations 1and 2 has been performed. This was done using the ginac symbolic algebra library(ref). Because of the complexity of the result, a one dimensional version was calculated in order to get a feeling for the form of the solution. The elements of the covariance matrix are shown in Equation 4. The derivatives of the elements are shown in Equation 12. The emittance squared is shown in Equation 23, and the derivative of this with respect to z is shown in 39. This is clearly not zero and the form of the equation provides a clue to the oscillating nature of the emittance. The only variable which varies along the channel is ρ. The derivative of the emittance is thus the sum of many oscillating terms.

$$
\begin{aligned}
C_{11} &= -<x_0>^2 - 2<r\sin(\alpha z+\phi_0)><x_0> + \qquad (4)\\
&<r^2\sin(\alpha z+\phi_0)^2> + 2<r\sin(\alpha z+\phi_0)x_0> + \qquad (5)\\
&<x_0^2> - <r\sin(\alpha z+\phi_0)>^2 \qquad (6)\\
C_{12} &= <\cos(\alpha z+\phi_0)r^2\sin(\alpha z+\phi_0)\alpha> - <r\sin(\alpha z+\phi_0)><\cos(\alpha z+\phi_0)r\alpha> + \qquad (7)\\
&<\cos(\alpha z+\phi_0)r\alpha x_0> - <x_0><\cos(\alpha z+\phi_0)r\alpha> \qquad (8)\\
C_{21} &= <\cos(\alpha z+\phi_0)r^2\sin(\alpha z+\phi_0)\alpha> - <r\sin(\alpha z+\phi_0)><\cos(\alpha z+\phi_0)r\alpha> + \qquad (9)\\
&<\cos(\alpha z+\phi_0)r\alpha x_0> - <x_0><\cos(\alpha z+\phi_0)r\alpha> \qquad (10)\\
C_{22} &= -<\cos(\alpha z+\phi_0)r\alpha>^2 + <\cos(\alpha z+\phi_0)^2r^2\alpha^2> \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
C'_{11} &= 2<\cos(\alpha z+\phi_0)r^2\sin(\alpha z+\phi_0)\alpha> - 2<r\sin(\alpha z+\phi_0)><\cos(\alpha z+\phi_0)r\alpha> + \qquad (12)\\
&2<\cos(\alpha z+\phi_0)r\alpha x_0> - 2<x_0><\cos(\alpha z+\phi_0)r\alpha> \qquad (13)\\
C'_{12} &= -<r\sin(\alpha z+\phi_0)\alpha^2x_0> - <r^2\sin(\alpha z+\phi_0)^2\alpha^2> - \qquad (14)\\
&<\cos(\alpha z+\phi_0)r\alpha>^2 + <r\sin(\alpha z+\phi_0)><r\sin(\alpha z+\phi_0)\alpha^2> + \qquad (15)\\
&<r\sin(\alpha z+\phi_0)\alpha^2><x_0> + <\cos(\alpha z+\phi_0)^2r^2\alpha^2> + \qquad (16)\\
&<\sin(\alpha z+\phi_0)> \qquad (17)\\
C'_{21} &= -<r\sin(\alpha z+\phi_0)\alpha^2x_0> - <r^2\sin(\alpha z+\phi_0)^2\alpha^2> - \qquad (18)\\
&<\cos(\alpha z+\phi_0)r\alpha>^2 + <r\sin(\alpha z+\phi_0)><r\sin(\alpha z+\phi_0)\alpha^2> + \qquad (19)\\
&<r\sin(\alpha z+\phi_0)\alpha^2><x_0> + <\cos(\alpha z+\phi_0)^2r^2\alpha^2> + \qquad (20)\\
&<\sin(\alpha z+\phi_0)> \qquad (21)\\
C'_{22} &= 2<r\sin(\alpha z+\phi_0)\alpha^2><\cos(\alpha z+\phi_0)r\alpha> - 2<\cos(\alpha z+\phi_0)r^2\sin(\alpha z+\phi_0)\alpha^3> \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
\epsilon^2 \quad = \quad & + - < r^2\cos(\rho)\sin(\rho)\alpha >^2 && (23)\\
& + 2 < r\sin(\rho) >< r\cos(\rho)\alpha >< r\cos(\rho)\alpha x_0 > && (24)\\
& + 2 < r^2\cos(\rho)\sin(\rho)\alpha >< r\cos(\rho)\alpha >< x_0 > && (25)\\
& + - < r^2\cos(\rho)^2\alpha^2 >< x_0 >^2 && (26)\\
& + 2 < r^2\cos(\rho)^2\alpha^2 >< r\sin(\rho)x_0 > && (27)\\
& + \ -2 < r\cos(\rho)\alpha >^2 < r\sin(\rho)x_0 > && (28)\\
& + 2 < r\cos(\rho)\alpha >< x_0 >< r\cos(\rho)\alpha x_0 > && (29)\\
& + 2 < r\sin(\rho) >< r^2\cos(\rho)\sin(\rho)\alpha >< r\cos(\rho)\alpha > && (30)\\
& + - < r\cos(\rho)\alpha x_0 >^2 && (31)\\
& + \ -2 < r^2\cos(\rho)\sin(\rho)\alpha >< r\cos(\rho)\alpha x_0 > && (32)\\
& + - < r^2\sin(\rho)^2 >< r\cos(\rho)\alpha >^2 && (33)\\
& + - < r\sin(\rho) >^2 < r^2\cos(\rho)^2\alpha^2 > && (34)\\
& + \ < r^2\sin(\rho)^2 >< r^2\cos(\rho)^2\alpha^2 > && (35)\\
& + \ -2 < r\sin(\rho) >< r^2\cos(\rho)^2\alpha^2 >< x_0 > && (36)\\
& + - < r\cos(\rho)\alpha >^2 < x_0^2 > && (37)\\
& + \ < r^2\cos(\rho)^2\alpha^2 >< x_0^2 > && (38)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\epsilon^2}{dz} &= +\,2 < r^2\cos(\rho)\sin(\rho)\alpha^3 >< x_0 >^2 && (39)\\
&+\;-4 < r^2\cos(\rho)\sin(\rho)\alpha^3 >< r\sin(\rho)x_0 > && (40)\\
&+\;-2 < r\sin(\rho) >< r^2\cos(\rho)\sin(\rho)\alpha >< r\sin(\rho)\alpha^2 > && (41)\\
&+\;-2 < r\sin(\rho) >< r\cos(\rho)\alpha x_0 >< r\sin(\rho)\alpha^2 > && (42)\\
&+\,4 < r\cos(\rho)\alpha >< r\sin(\rho)x_0 >< r\sin(\rho)\alpha^2 > && (43)\\
&+\;-2 < r^2\cos(\rho)\sin(\rho)\alpha >< x_0 >< r\sin(\rho)\alpha^2 > && (44)\\
&+\;-2 < r^2\cos(\rho)\sin(\rho)\alpha >< \sin(\rho) > && (45)\\
&+\;-2 < r\cos(\rho)\alpha >< x_0 >< r^2\sin(\rho)^2\alpha^2 > && (46)\\
&+\;-2 < r\cos(\rho)\alpha x_0 >< \sin(\rho) > && (47)\\
&+\;-2 < r\sin(\rho) >< r\cos(\rho)\alpha >< r^2\sin(\rho)^2\alpha^2 > && (48)\\
&+\,2 < r^2\sin(\rho)^2 >< r\cos(\rho)\alpha >< r\sin(\rho)\alpha^2 > && (49)\\
&+\,2 < r^2\cos(\rho)\sin(\rho)\alpha >< r\sin(\rho)\alpha^2 x_0 > && (50)\\
&+\,2 < r\cos(\rho)\alpha x_0 >< r\sin(\rho)\alpha^2 x_0 > && (51)\\
&+\,2 < r\cos(\rho)\alpha >< x_0 >< \sin(\rho) > && (52)\\
&+\,2 < r\cos(\rho)\alpha x_0 >< r^2\sin(\rho)^2\alpha^2 > && (53)\\
&+\;-2 < r\sin(\rho) >< r\cos(\rho)\alpha >< r\sin(\rho)\alpha^2 x_0 > && (54)\\
&+\,2 < r^2\cos(\rho)\sin(\rho)\alpha >< r^2\sin(\rho)^2\alpha^2 > && (55)\\
&+\;-2 < r\cos(\rho)\alpha >< x_0 >< r\sin(\rho)\alpha^2 x_0 > && (56)\\
&+\,2 < r\cos(\rho)\alpha >< x_0^2 >< r\sin(\rho)\alpha^2 > && (57)\\
&+\,2 < r\sin(\rho) >< r\cos(\rho)\alpha >< \sin(\rho) > && (58)\\
&+\,4 < r\sin(\rho) >< r^2\cos(\rho)\sin(\rho)\alpha^3 >< x_0 > && (59)\\
&+\;-2 < r^2\cos(\rho)\sin(\rho)\alpha^3 >< x_0^2 > && (60)\\
&+\;-2 < r^2\sin(\rho)^2 >< r^2\cos(\rho)\sin(\rho)\alpha^3 > && (61)\\
&+\,2 < r\sin(\rho) >^2< r^2\cos(\rho)\sin(\rho)\alpha^3 > && (62)\\
&+\;-2 < x_0 >< r\cos(\rho)\alpha x_0 >< r\sin(\rho)\alpha^2 > && (63)
\end{aligned}
$$

6 Conclusions

An oscillation has been observed in the RMS emittance of a beam propagating in a uniform solenoidal field. This seems to be a statistical effect, but this has not been proven. The derivative of the RMS emittance with respect to z has been calculated analytically for the two dimensional case and is not identically zero. If the effect is statistical in nature, this should be identically zero in the limit of infinite statistics, but this has not been investigated. An attempt has been made to calculate the derivative of the four dimensional RMS emittance; owing to the complexity of the result, a cross check needs to be performed before a conclusion can be drawn. In general, because both the monte carlo and analytical results have not been cross checked by independent analysis no conclusions should be drawn. There is an urgent need to do these cross checks, because of the possible impact of this effect.

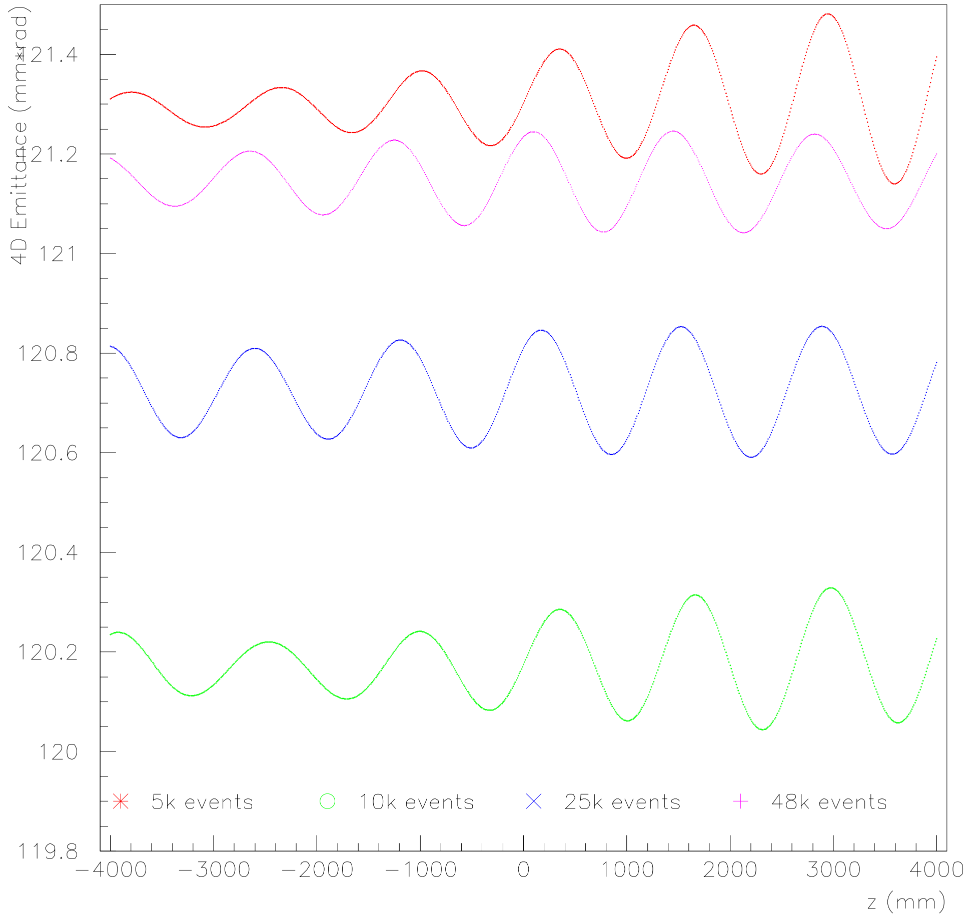

Figure 1: Emittance as a function of z for a xxx GeV/c beam and different sample sizes.

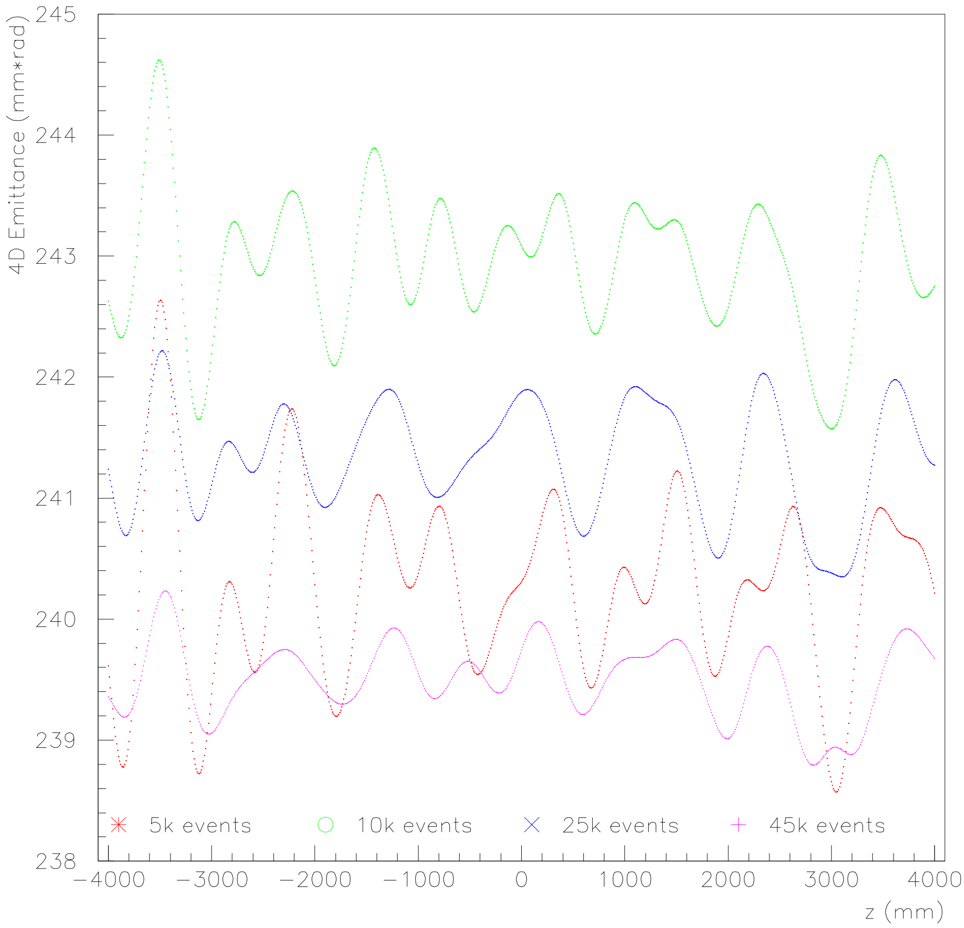

Figure 2: Emittance as a function of z for a xxx to yyy GeV/c beam and different sample sizes.

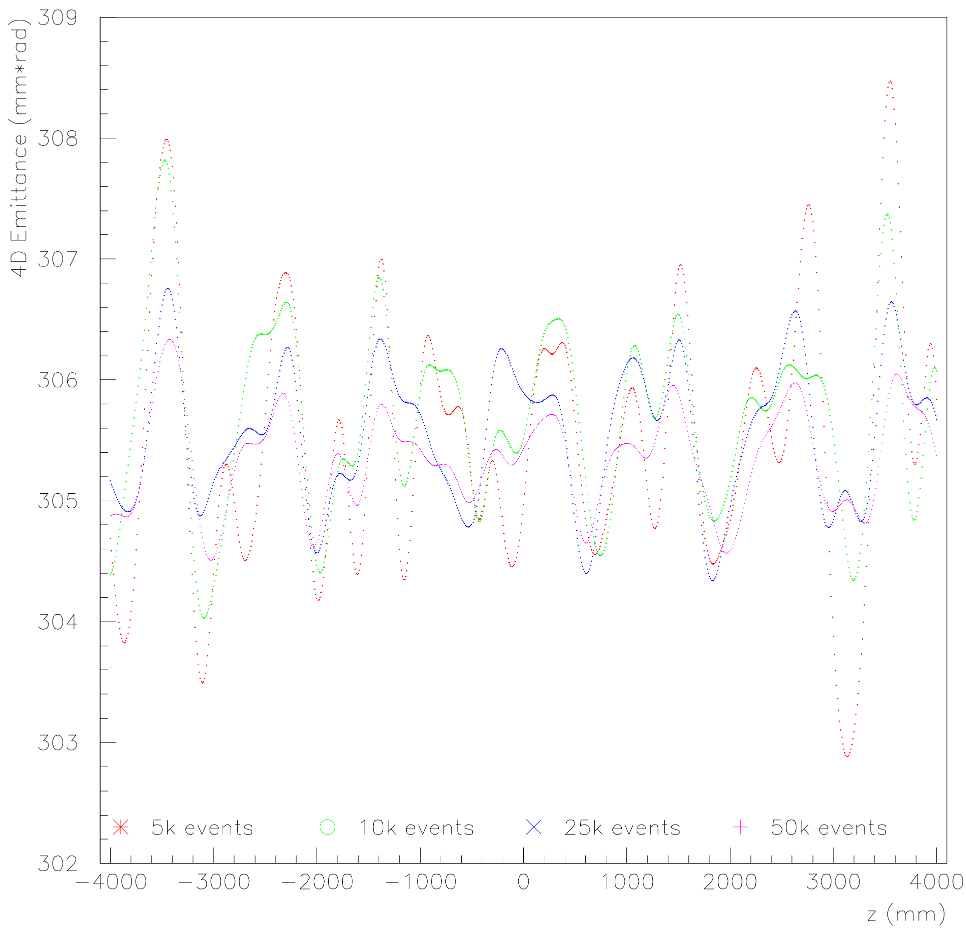

Figure 3: Emittance as a function of z for a xxx to yyy GeV/c beam with no event selection.